

Lorentz Invariant Superluminal Tunneling

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It is shown that superluminal optical signalling is possible without violating Lorentz invariance and causality via tunneling through photonic band gaps in inhomogeneous dielectrics of a special kind.

I. INTRODUCTION

A number of recent experiments have reported the observation of electromagnetic waves propagating with velocities larger than c (the velocity of light in vacuum) in dispersive media [1], wave guides [2], electronic circuits [3] and in tunneling [4]. The experimenters have been quick to point out that these observations do not necessarily contradict the special theory of relativity and causality. These claims have naturally generated a controversy in the literature.

In the case of dispersive propagation the claim is in apparent contradiction with the pioneering work of Sommerfeld and Brillouin [5] who clearly showed the difference between group velocity, phase velocity and signal front velocity, and established the result that no physical signal can travel faster than c in dispersive media. However, it has recently been argued that for physical signals that are of finite duration the causality principle “cause precedes effect” is preserved despite superluminal motion. This is because a superluminal signal travelling backward in time can never arrive before the primary signal is generated, thus preventing the original user changing the transmitted signal [6].

In the case of frustrated total internal reflection (FTIR) and tunneling, the situation is quite different. It has been argued that in such cases the wave number is imaginary, the phase is a constant and the concept of a signal front is meaningless [6]. Further, it has been pointed out that if the signal is narrow-band limited, there is no distortion of the signal envelope and its delay is the same as that of its centre of gravity [7]. Since the evanescent (exponentially damped) component of a wave does not oscillate with distance, it does not accumulate any phase and can therefore propagate through the evanescent region with zero (phase) delay. It has been argued that there is empirical evidence of this in, for example, symmetrical FTIR in which there is no time lag between the reflected and tunneled signals [6]. However, it is not quite clear how a zero phase delay necessarily implies a zero signal delay.

One source of confusion in the literature, in our opinion, is the popular use of an analogy between the Helmholtz and Schrödinger equations. Since Maxwell’s equations in an inhomogeneous but isotropic medium reduce to the Helmholtz equation for a monochromatic wave in the scalar approximation, and the Helmholtz and the *non-relativistic* Schrödinger equations are formally identical, the one-dimensional process of non-relativistic quantum mechanical tunneling has been used to model the optical process of transmission through a barrier [6]. This is obviously unsatisfactory, because (a) the Schrödinger evolution used is characteristically non-relativistic whereas the optical processes in question are intrinsically relativistic, and (b) the Helmholtz function for the electric field is real whereas the Schrödinger wave function is complex. It would therefore be preferable to use a reliable and consistent quantum mechanical formalism for photons.

Fortunately, such a formalism exists [8], and is based on the classic works of Kemmer [9] and Harish-Chandra [10]. In this formalism, the wave function for the photon, which obeys a first-order equation similar to the Dirac equation, is a ten component column whose first six elements (the electric and magnetic field strengths) are real functions and the last four are zero, and there is a conserved four-vector current associated with energy flow (not charge flow as in the familiar case of charged particles with a complex wave function) whose time component is positive definite and can be interpreted as a probability density. The phase of such a wave function is obviously not expressible as a multiplicative exponential factor but is rather given in the same way as in classical electrodynamics through an additive term in the sinusoidal function for the fields. The signal velocity can be calculated in this formalism unambiguously from the energy flux vector which turns out to be proportional to the Poynting vector, as one would expect.

It is the purpose of this paper to show, using this formalism, that Einstein causal electromagnetic signals *can*

indeed travel faster than c while tunneling through a photonic band gap provided that the dielectric in the gap is inhomogeneous and (practically) non-dispersive. The same result will be shown to hold for classical light.

II. THE TUNNELING SOLUTION IN ELECTRODYNAMICS

Let us consider the usual tunneling problem with a thin non-magnetic, practically non-absorptive material with a band gap around the frequency ω , extending from $x = 0$ to $x = d$ and the signal incident normally on it so that there is no dispersion. It is essentially a two-dimensional problem (in the $x - y$ plane) expressible in terms of a single component of the electric or magnetic field [12]. We will consider the case of electric polarization with $H_x = H_z = 0$, $E_x = E_y = 0$ and $\mu = 0$, $\epsilon = \epsilon(x)$, $\epsilon_0 = 1$. The same result will hold for magnetic polarization also. Then Maxwell's equations can be written in the rest frame of the dielectric material in the form

$$\partial_y E_z = 0, \quad \partial_x E_z = \frac{1}{c} \partial_t H_y \quad (1)$$

$$\partial_z H_y = 0, \quad \partial_x H_y = \frac{\epsilon(x)}{c} \partial_t E_z \quad (2)$$

$$\partial_x^2 E_z - \frac{\epsilon(x)}{c^2} \partial_t^2 E_z = 0 \quad (3)$$

$$\partial_x^2 H_y + \partial_y^2 H_y + (\partial_x \ln \epsilon(x)) \partial_x H_y - \frac{\epsilon(x)}{c^2} \partial_t^2 H_y = 0 \quad (4)$$

Let us first assume that the time variation of the electric and magnetic fields is given by $\exp(\pm i\omega t)$, and use the ansatz $E_z(x, y) = Y(x)U(y)$. Then it is easy to show that

$$U(y) = \beta e^{\pm i \frac{\omega}{c} \alpha y} \quad (5)$$

where β and α are constants. It follows from (1) that $\alpha = 0$, and so we have

$$E_z = \beta Y(x) e^{\pm i\omega t} \quad (6)$$

$$H_y = \frac{\mp i c \beta}{\omega} \frac{dY(x)}{dx} e^{\pm i\omega t} \quad (7)$$

This shows that the magnetic field H_y is completely determined by the electric field E_z . It also follows from (1) and (2) that

$$\frac{dY(x)}{dx} = -\frac{\omega^2}{c^2} \int \epsilon(x) Y(x) dx \quad (8)$$

or,

$$\frac{d^2 Y(x)}{dx^2} + \frac{\omega^2}{c^2} \epsilon(x) Y(x) = 0 \quad (9)$$

An approximate solution to this equation (9) is given by

$$Y(x) \approx [k(x)]^{-\frac{1}{2}} [c_1 \exp[-i \int_0^x k(x) dx] + c_2 \exp[i \int_0^x k(x) dx]] \quad (10)$$

where $k = \sqrt{\epsilon(x)}\omega/c$, c_1 and c_2 are arbitrary constants, and we have assumed that the change in $\epsilon(x)$ over one wavelength ($2\pi/k$) is sufficiently small compared to $|\epsilon(x)|$ (WKB approximation). This gives the usual oscillating solution of $E_z(x, t)$:

$$E_z(x, t) \approx [k(x)]^{-\frac{1}{2}} [c_1 \exp[-i(\int_0^x k(x) dx - \omega t)] + c_2 \exp[i(\int_0^x k(x) dx - \omega t)]] \quad (11)$$

Since the dielectric has a band gap around the frequency ω , these oscillating solutions cannot propagate through it. One has to look for exponential or tunneling solutions. In the case of the non-relativistic Schrödinger equation such solutions are obtained when the function corresponding to $\epsilon(x)$, namely, $[E - V(x)]$, becomes negative. This is not possible in electrodynamics because $\epsilon(x)$ is never negative. However, it is significant that a general tunneling solution can still be found, and is given by

$$E_z^d(x, t) \approx [\kappa(x)]^{-\frac{1}{2}} [c_1 \exp[-\int_0^x \kappa(x) dx + \omega t] + c_2 \exp[\int_0^x \kappa(x) dx - \omega t]] \quad (12)$$

with $\kappa(x) = \omega \sqrt{\epsilon(-ix)}/c$ a real, positive function [13]. This is clearly a solution of the wave equation

$$\partial_x^2 E_z^d - \frac{\epsilon(-ix)}{c^2} \partial_t^2 E_z^d = 0 \quad (13)$$

which is Lorentz invariant as long as $\epsilon(-ix)$ is a real, positive Lorentz scalar function. That is guaranteed if $\epsilon(x, t)$ is a real, positive definite function of the Lorentz invariant variable $(x^2 - c^2 t^2)$ in an arbitrary inertial frame. We will therefore restrict our discussions to such cases only.

Notice that the tunneling solution (12) is a mapping of the oscillating solution (11) by

$$x \rightarrow -ix, \quad t \rightarrow -it \quad (14)$$

Maxwell's equations in vacuo are invariant under this mapping. Maxwell's equations in an inhomogeneous dielectric [equations (1) - (4)] are also invariant provided $\epsilon(-ix) = \epsilon(x)$. But that is certainly not the most general case. Assuming that $\epsilon(x)$ is an analytic function, one can express it as a Taylor series around $x = 0$:

$$\epsilon(x) = \epsilon_0 + \sum_n a_n x^n \quad (15)$$

with the sum positive definite [5]. Thus $\epsilon(-ix)$ will be complex in general. But, since $\text{Im} \sqrt{\epsilon(-ix)}$ will give rise to oscillating terms in (12), and since the material is assumed to have a band gap around ω , it must vanish. Maxwell's equations then get mapped on to equations, such as equation (13), that are still Lorentz invariant and therefore acceptable. It is clear from equation (13) that the propagation will be superluminal provided $\epsilon(-ix) < \epsilon_0 (= 1)$. This is possible, for example, if the dielectric function $\epsilon(-ix) = (1 + \sum_n a_n x^n) < 1$ with n such that $\text{Im} \sqrt{\epsilon(-ix)} = 0$ and $\sum_n a_n x^n < 0$.

An immediate consequence of the mapping (14) is that time-like intervals are mapped on to space-like intervals $(c^2 t^2 - x^2) \rightarrow (x^2 - c^2 t^2)$. Consequently, if $\epsilon(-ix) < 1$, *all causally related events get connected by superluminal signals*. Conversely, it is straightforward to see that superluminal signals ($v > c$) imply the mapping (14), because

$$\begin{aligned} x' &= (x - vt)/\sqrt{1 - v^2/c^2} = -i(x - vt)/\sqrt{v^2/c^2 - 1} \\ t' &= (t - vx/c^2)/\sqrt{1 - v^2/c^2} = -i(t - vx/c^2)/\sqrt{v^2/c^2 - 1} \end{aligned} \quad (16)$$

This is remarkable and important for the interpretation of the experiments showing superluminal tunneling— they do not contradict Lorentz invariance and causality.

It is instructive to look at the difference between superluminal optical tunneling and tunneling of massive particles. While tunneling, the energy and momentum of massive relativistic particles are imaginary, as one can easily verify by applying the energy and momentum operators on their wavefunction. Thus, the relativistic relation $E^2 = p^2 c^2 + m_0^2 c^4$ gets mapped on to $E^2 = p^2 c^2 - m_0^2 c^4$, implying tachyons. This does not happen for massless bosons. Nevertheless, as we have seen above, tunneling solutions in electrodynamics are also superluminal.

It is often asserted that according to the special principle of relativity the maximum velocity that a physical signal can have is the velocity of light c in vacuum. If that is correct, then the special relativity principle would rule out the possibility of dielectric materials of the kind discussed above. That would imply that somehow only dielectrics with the property $\epsilon(-ix) = \epsilon(x)$ can exist physically. Whereas that is not impossible, we find it hard to believe that such

a demonstration can indeed be given. On the other hand, if one restricts oneself to the assumptions actually made by Einstein, namely the postulate of relativity of uniform motion coupled with the postulate that the velocity of light is independent of the motion of the light source, one need only insist on Lorentz invariance as a necessary condition for a physical law [14]. That would leave open the possibility of dielectrics of the kind that would make superluminal yet causal signals possible in tunneling modes.

Interestingly, the dielectrics chosen in the tunneling experiments [4] all had variable layers of dielectrics and were practically dispersion free. Now, it is well-known that causality and dispersion relations are intimately related [11]. It follows from these dispersion relations that the real part of the refractive index n must vanish for a purely non-dispersive material. Hence the velocity of propagation c/n of light through such a material has no upper limit. The problem is to produce such materials. The trick is to prepare a medium in such a way that it is inhomogeneous with alternative thin layers of high and low refractive indices n_i that are all greater than unity ($n_i > 1$) so that it acquires a photonic band gap. Then the evanescent wave sees a refractive index < 1 , as we have seen, and so propagates superluminally without changing shape.

III. QUANTUM MECHANICAL FORMULATION OF OPTICAL TUNNELING

We will now show how to give a purely quantum mechanical formulation of this superluminal tunneling behaviour. For this we need to use a consistent quantum mechanical formulation of massless electrodynamics using the Kemmer–Harish-Chandra formalism [8], outlined in the Appendices. It is clear from this formalism that the classical Maxwell fields are components of a ten-component quantum mechanical wavefunction with constraints that reduce the degrees of freedom to two. For the tunneling problem, the number of degrees of freedom is further reduced to one, as we have already seen. Let the incident finite duration signal be represented by the electric fields (components of the ten dimensional unnormalized photon wavefunction $\gamma\psi$, vide Appendix A)

$$E_z^i = \int dk A(k) \cos(kx - \omega t - \phi) - \sqrt{R} \int dk A(k) \cos(kx + \omega t + \phi) \quad \text{for } x \leq 0 \quad (17)$$

$$E_z^d = \theta(t) \frac{1}{\sqrt{\kappa(x)}} C \exp\left[-\int_0^x \kappa(x) dx + \omega_0 t\right] \quad \text{for } 0 \leq x \leq d \quad (18)$$

$$E_z^f = \theta(t - \tau) \sqrt{T} \int dk A(k) \cos[k(x - d) - \omega(t - \tau) + \chi] \quad \text{for } x \geq d \quad (19)$$

where $A(k) = (1/\sqrt{2\pi\sigma^2}) \exp[-(k - k_0)^2/2\sigma^2]$ is real and $\int_{-\infty}^{\infty} A(k) dk = 1$, $\int_{-\infty}^{\infty} k A(k) dk = k_0$. R and T are the reflection and transmission coefficients, $k = \omega/c$, $\kappa(x) = k_0 \sqrt{\epsilon(-ix)}$, τ is the tunneling or dwell time and $\theta(t)$ is the step function. (Note that there is no term representing a reflected wavefunction within the tunneling region because we are not considering a steady state situation or times $t > \tau$.) Accordingly, the dielectric medium is at rest (in the sense of being free of any disturbance) before $t = 0$ and there is no emerging signal at $x = d$ before $t = \tau$. By matching the wavefunctions smoothly at the boundary $x = 0$, $t = 0$, we get

$$C = \sqrt{\kappa(0)}(1 - \sqrt{R}) \cos \phi \quad (20)$$

$$\tan \phi = \frac{\kappa(0)}{k_0} = 1 \quad (21)$$

Hence

$$E_z^d = \theta(t) \frac{\sqrt{\kappa(0)}(1 - \sqrt{R}) \cos \phi}{\sqrt{\kappa(x)}} \exp\left[-\int_0^x \kappa(x) dx + \omega_0 t\right] \quad (22)$$

The magnetic field in the tunneling region is determined by the analog of (7) for the tunneling case and is given by

$$H_y^d = \theta(t) \frac{c}{\omega_0} \partial_x E_z^d \quad (23)$$

Therefore we have (in the WKB approximation)

$$H_y^d = -\theta(t) \sqrt{\kappa(0)} (1 - \sqrt{R}) \sqrt{\kappa(x)} \cos \phi \frac{c}{\omega_0} \exp[-\int_0^x \kappa(x) dx + \omega_0 t] \quad (24)$$

Matching the wavefunctions at the other boundary $x = d$, $t = \tau$ gives

$$\sqrt{T} = \frac{\sqrt{\kappa(0)} (1 - \sqrt{R}) \cos \phi}{\sqrt{\kappa(d)}} \sec \chi \exp[-\int_0^d \kappa(x) dx + \omega_0 \tau] \quad (25)$$

Further, matching the derivatives of the wavefunctions at this boundary, one has

$$\tan \chi = \frac{\kappa(d)}{k_0} \quad (26)$$

The velocity operator in this formalism is the 10×10 matrix $v\tilde{\beta}_x = (c/\sqrt{\epsilon(-ix)})$ ($\beta_0\beta_x - \beta_x\beta_0$). Thus the Poynting vector can now be calculated, and is given by (see Appendix A)

$$\begin{aligned} S_x^d &= m_0 c^3 \psi^T \gamma \tilde{\beta}_x \gamma \psi = -c E_z^d H_y^d \\ &= \theta(t) \kappa(0) (1 - \sqrt{R})^2 \cos^2 \phi \frac{c^2}{2\omega_0} \exp[-2(\int_0^x \kappa(x) dx - \omega_0 t)] \end{aligned} \quad (27)$$

The energy density is given by (see Appendix A)

$$\begin{aligned} \mathcal{E}^d &= \frac{1}{2} \psi^T \gamma \psi = \frac{1}{2} [\epsilon(-ix) E_z^{d2} + H_y^{d2}] \\ &= \theta(t) \kappa(0) (1 - \sqrt{R})^2 \cos^2 \phi \frac{c^2}{2\omega_0^2} \kappa(x) \exp[-2(\int_0^x \kappa(x) dx - \omega_0 t)] \end{aligned} \quad (28)$$

One can therefore calculate the velocity of energy transport

$$v_x^d = \frac{S_x}{\mathcal{E}^d} = \frac{c}{\sqrt{\epsilon(-ix)}} \quad (29)$$

It follows from this that the tunneling time is given by

$$\tau = \int_0^d \frac{dx}{v_x^d} \quad (30)$$

which implies

$$\int_0^d \kappa(x) dx - \omega_0 \tau = 0 \quad (31)$$

In a hypothetical model in which $\sqrt{\epsilon(-ix)} = 1 - ax^2$,

$$\tau = \frac{d}{c} - \frac{ad^3}{3c} \quad (32)$$

which is always less than the time for passage through vacuum. This superluminal effect will be further accentuated if one includes higher order terms in x in the expansion of $\sqrt{\epsilon(-ix)}$ because of the condition $\sum_n a_n x^n < 0$ stated above.

If one uses the de Broglie-Bohm guidance condition $v_x^d = dx/dt$, one again obtains the same result for τ . These results confirm that the energy and so the physical signal indeed propagates superluminally while tunneling.

IV. CONCLUSIONS

In conclusion we would like to emphasize precisely the significant new result that we have obtained. Since there has been much discussion and some controversy in the literature regarding superluminal effects and their causality, let us summarize the situation as we see it.

The materials used for observing superluminal effects have been generally termed “ultrarefractive” [15]. Near the edges of a transmission gap the effective permittivity can become close to zero. Consequently, surprising effects can be observed on light transmitted and reflected by such materials, such as superluminal velocities as well as enlargement and splitting of the transmitted beam.

In one type of process the effects are results of anomalous dispersion, i.e., anomalous variation of the permittivity with wavelength. Although the 1914 analysis of Sommerfeld and Brillouin clearly established that superluminality in such cases cannot be Einstein causal and is only apparent, it has recently been argued that this need not be the case for physical signals that are of finite duration and extent because a responsive signal travelling backward in time in such a case cannot arrive before the primary signal is generated, thus preserving the causality principle [6]. Our paper does not deal with this type of phenomena.

The second type of process involves tunneling in one (or two) dimensions through a narrow band gap, and it is only this type of phenomena (1D tunneling) that we have addressed. The theoretical discussions of such phenomena have so far been based purely on an analogy between the non-relativistic Schrödinger equation and the Helmholtz equation leading to an effective refractive index $n(x, y, z) = \{2m[E - V(x, y, z)]\}^{1/2}c/\hbar\omega$ which is imaginary in any region where $E < V$ [4,16]. This mechanism is, in reality, not applicable to photons, as we have mentioned earlier and as Chiao and Steinberg admit in their review article [16]. To take a definite stand on an issue such as superluminal propagation and causality, analogies are not reliable in our opinion, and one must use a proper theory, namely a consistent relativistic quantum mechanical formalism for photons [8]. We have used this formalism to carry out explicit calculations for the tunneling of a finite width photon wave-packet incident normally on a 1D photonic barrier. (Note that in this sense also our result is new because total internal reflection in optics occurs only for *non-zero* critical angles of incidence.) Our analysis clearly shows that genuine Einstein causal superluminal propagation can occur only if the tunneling medium is inhomogeneous on the scale of the wavelength and $\text{Im } \epsilon(-ix) = 0$. This follows simply and very generally from the fact that points on the light cone remain on the light cone under the mapping (14) which takes propagating solutions to tunneling solutions. Therefore, the only way to get genuine superluminal signals is to have an inhomogeneous dielectric function $\epsilon(x) > 1$ that is mapped to $\epsilon(-ix) < 1$ with $\text{Im } \epsilon(-ix) = 0$ to ensure Lorentz invariance of the wave equation (13). This argument obviously holds for both classical and quantum light, and is consistent with dispersion relations and causality [11].

Such materials have been used in actual experiments [16,17]. They involve tunneling at near normal incidence through band gaps excited in periodic dielectric structures. These band gaps arise from Bragg reflections from the periodic structure, leading to an evanescent decay of the wave amplitude when the frequency is within the forbidden band gap at the first Brillouin zone. It should be noted that such periodic structures are *non-dispersive* so that the tunneling wave-packets that are tuned to midgap remain essentially undistorted upon transmission through the barrier, though much attenuated in amplitude [16].

V. ACKNOWLEDGEMENT

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VI. APPENDIX A

Until recently, no consistent quantum mechanical formalism existed for relativistic bosons below the threshold for pair production and annihilation. Relativistic quantum mechanics can only be consistently formulated provided there

exists a conserved four-vector current whose time component, to be identified with the probability density, is positive definite. Unfortunately, the conserved charge vector current for relativistic spin 0 and spin 1 bosons does not have this property. Moreover, the charge current vanishes for neutral particles like the photon. However, it has now been shown [8] that a conserved four-vector current with a positive definite time component does exist for relativistic bosons, and is associated, not with the charge current but, with the flow of energy. This formulation is based on the first-order Kemmer equation [9]

$$(i \hbar \beta_\mu \partial^\mu + m_0 c) \psi = 0 \quad (33)$$

where the matrices β satisfy the algebra

$$\beta_\mu \beta_\nu \beta_\lambda + \beta_\lambda \beta_\nu \beta_\mu = \beta_\mu g_{\nu\lambda} + \beta_\lambda g_{\nu\mu}. \quad (34)$$

The 5×5 dimensional representation of these matrices describes spin 0 bosons and the 10×10 dimensional representation describes spin 1 bosons. Multiplying (33) by β_0 , one obtains the Schrödinger form of the equation

$$i \hbar \frac{\partial \psi}{\partial t} = [-i \hbar c \tilde{\beta}_i \partial_i - m_0 c^2 \beta_0] \psi \quad (35)$$

where $\tilde{\beta}_i \equiv \beta_0 \beta_i - \beta_i \beta_0$. Multiplying (33) by $1 - \beta_0^2$, one obtains the first class constraint

$$i \hbar \beta_i \beta_0^2 \partial_i \psi = -m_0 c (1 - \beta_0^2) \psi. \quad (36)$$

It implies the conditions $\text{div} \vec{D} = -(m_0^2 c / \hbar) A_0$ and $\vec{B} = \text{curl} \vec{A}$ if one takes

$$\psi^T = (1/\sqrt{m_0 c^2})(-D_x, -D_y, -D_z, B_x, B_y, B_z, -m_0 A_x, -m_0 A_y, -m_0 A_z, m A_0) \quad (37)$$

The reader is referred to Ref. [8] for further discussions regarding the significance of this constraint.

If one multiplies equation (35) by ψ^\dagger from the left, its hermitian conjugate by ψ from the right and adds the resultant equations, one obtains the continuity equation

$$\frac{\partial (\psi^\dagger \psi)}{\partial t} + \partial_i \psi^\dagger \tilde{\beta}_i \psi = 0. \quad (38)$$

This can be written in the form

$$\partial^\mu \Theta_{\mu 0} = 0 \quad (39)$$

where

$$\Theta_{\mu\nu} = -m_0 c^2 \bar{\psi} (\beta_\mu \beta_\nu + \beta_\nu \beta_\mu - g_{\mu\nu}) \psi \quad (40)$$

(with $\bar{\psi} = \psi^\dagger \eta_0$, $\eta_0 = 2\beta_0^2 - 1$, $\eta_0^2 = 1$) is the symmetric energy-momentum tensor, and

$$\Theta_{00} = -m_0 c^2 \psi^\dagger \psi < 0 \quad (41)$$

Thus, it is possible to define a wave function $\phi = \sqrt{m_0 c^2 / E} \psi$ (with $E = -\int \Theta_{00} dV$) such that $\phi^\dagger \phi$ is non-negative and normalized and can be interpreted as a probability density. The conserved probability current density is $s_\mu = -\Theta_{\mu 0} / E = (\phi^\dagger \phi, -\phi^\dagger \tilde{\beta}_i \phi)$.

Notice that according to the equation of motion (35), the velocity operator for massive bosons is $c \tilde{\beta}_i$.

The theory of massless spin 0 and spin 1 bosons cannot be obtained simply by taking the limit m_0 going to zero because of the $1/\sqrt{m_0}$ factor in ψ . One has to start with the equation [10]

$$i \hbar \beta_\mu \partial^\mu \psi + m_0 c \gamma \psi = 0 \quad (42)$$

where γ is a matrix that satisfies the following conditions:

$$\gamma^2 = \gamma \quad (43)$$

$$\gamma \beta_\mu + \beta_\mu \gamma = \beta_\mu. \quad (44)$$

This equation can be derived from the gauge invariant Lagrangian density

$$\mathcal{L} = -\frac{i\hbar}{2}[\partial^\mu \bar{\psi} \gamma \beta_\mu \psi - \bar{\psi} \beta_\mu \gamma \partial^\mu \psi] + \frac{m_0 c}{2} \bar{\psi} \gamma \psi \quad (45)$$

Multiplying (42) from the left by $1 - \gamma$, one obtains

$$\beta_\mu \partial^\mu (\gamma \psi) = 0. \quad (46)$$

Multiplying (42) from the left by $\partial_\lambda \beta^\lambda \beta^\nu$, one also obtains

$$\partial^\lambda \beta_\lambda \beta_\nu (\gamma \psi) = \partial_\nu (\gamma \psi). \quad (47)$$

It follows from (46) and (47) that

$$\square (\gamma \psi) = 0 \quad (48)$$

which shows that $\gamma \psi$ describes massless bosons.

The Schrödinger form of the equation

$$i \hbar \frac{\partial (\gamma \psi)}{\partial t} = -i \hbar c \tilde{\beta}_i \partial_i (\gamma \psi) \quad (49)$$

and the associated first class constraint

$$i \hbar \beta_i \beta_0^2 \partial_i \psi + m_0 c (1 - \beta_0^2) \gamma \psi = 0 \quad (50)$$

follow by multiplying (42) by β_0 and $1 - \beta_0^2$ respectively. Equation (49) implies the Maxwell equations $\text{curl} \vec{E} = -(\mu/c) \partial_t \vec{H}$ and $\text{curl} \vec{H} = (\epsilon/c) \partial_t \vec{E}$ if

$$\gamma \psi^T = (1/\sqrt{m_0 c^2})(-D_x, -D_y, -D_z, B_x, B_y, B_z, 0, 0, 0, 0) \quad (51)$$

The constraint (50) implies the relations $\text{div} \vec{E} = 0$ and $\vec{B} = \text{curl} \vec{A}$. The symmetrical energy-momentum tensor is

$$\Theta_{\mu\nu} = -\frac{m_0 c^2}{2} \bar{\psi} (\beta_\mu \beta_\nu + \beta_\nu \beta_\mu - g_{\mu\nu}) \gamma \psi \quad (52)$$

and so the energy density

$$\mathcal{E} = -\Theta_{00} = \frac{m_0 c^2}{2} \psi^\dagger \gamma \psi = \frac{1}{2} [\vec{E} \cdot \vec{E} + \vec{B} \cdot \vec{B}] \quad (53)$$

is positive definite. The rest of the arguments are analogous to the massive case.

The Bohmian 3-velocity v_i for massless bosons can be defined by

$$v_i = c \frac{\psi^T \gamma \tilde{\beta}_i \gamma \psi}{\psi^T \gamma \psi} \quad (54)$$

Notice that in relativistic quantum mechanics the Bohmian velocity is not defined through the gradient of the phase as in non-relativistic quantum mechanics but in terms of the energy flux current.

Neutral massless vector bosons are very special in quantum mechanics. Their wave function is real, and so their charge current $j_\mu = \psi^T \beta_\mu \gamma \psi$ vanishes. However, their probability current density s_μ does not vanish. Furthermore, the Poynting vector turns out to be given by

$$S_i = m_0 c^3 \psi^T \gamma \tilde{\beta}_i \gamma \psi = c [\vec{E} \times \vec{H}]_i \quad (55)$$

One might wonder about the significance of the mass parameter m_0 for massless electrodynamics. It is necessary for a consistent quantum mechanical formalism for dimensional reasons and drops out of all physical results because of the operator γ . It can be altogether eliminated in favour of the intrinsic parameters in the theory, namely c , \hbar , the frequency ω and the spin multiplicity s .

The representations of the Kemmer-Duffin-Petiau β matrices used in this paper are given in Appendix B.

VII. APPENDIX B

$$\begin{aligned}
 i\beta_1 &= \begin{pmatrix} 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & -1 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & -1 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 1 & 0 & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 1 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & -1 & 0 & . & 0 & 0 & 0 & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ -1 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \end{pmatrix} & i\beta_2 = \begin{pmatrix} 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & -1 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 1 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & -1 & 0 & 0 & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & 0 & 0 & -1 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 1 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & -1 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \end{pmatrix} \\
i\beta_3 &= \begin{pmatrix} 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & -1 \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & -1 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 1 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & 0 & 1 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & -1 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & -1 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \end{pmatrix} & \beta_0 = \begin{pmatrix} 0 & 0 & 0 & . & 0 & 0 & 0 & . & -i & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & -i & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & -i & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ i & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & i & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ 0 & 0 & i & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & 0 & . & 0 \end{pmatrix}
\end{aligned}$$

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